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Bremsstrahlung for $p\bar{p}$ Production of Charmonium

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ABSTRACT

Radiation processes affecting the lineshapes of charmonium resonances produced in $p\bar{p}$ annihilation are considered. It is shown that only the soft infrared-enhanced radiation is important and that the $p\bar{p}$ case is simpler than the e^+e^- case, taking proper account of the proton's larger mass.

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Recent experiments have studied the production of the low-lying $c\bar{c}$ bound resonances (the ψ, ψ', χ_1, χ_2 charmonium states) in s-channel proton-antiproton annihilation.[1] As in the case of charmonium production from e^+e^- annihilation,[2] proper treatment of the resonance lineshapes requires taking into account the effects of real and virtual photon emission and finite beam energy resolution, which distort the underlying Breit-Wigner shapes by redistributing the initial energy $\sqrt{s}$. Here we give an account only of the bremsstrahlung. The relevant energy scales are the proton mass m_p, the energy $\sqrt{s}$, and the resonance masses $M_R \simeq$ 3000-3700 MeV and widths $\Gamma_R \lesssim$ few MeV. Our primary interest lies in the shape, not the normalization, of the cross sections, so only radiation from the annihilation vertex and possibly from the interference of the initial state with charged final-state charmonium decay products is important. The scale for radiated photon energy is set by the resonance widths. The differences from the e^+e^- case are the proton's larger mass, the (anti)proton being only mildly relativistic at these energies ($v/c \simeq 0.8$), and the proton's composite structure, which turns out to be irrelevant.

For $s \to M_R$, the Breit-Wigner resonance cross section is:

$$\sigma_{BW}(s) = \frac{\sigma_0 \cdot \Gamma_R^2/4}{(\sqrt{s} - M_R)^2 + \Gamma_R^2/4}. \tag{1}$$

The uncorrected peak cross section σ_0 is proportional to the product of partial widths $\Gamma(R \to p\bar{p})$ and $\Gamma(R \to final)$. The effect of one-photon radiation from the p and $\bar{p}$ lines on the lineshape is included by integrating over the standard spectrum to produce the corrected cross section $\sigma(s)$:[2,3]

$$\begin{aligned} \sigma(s) = \beta \int_0^{\sqrt{s}/2} \frac{dk}{k} \sigma_{BW}(s - 2k\sqrt{s}) \\ - 2\beta \int_0^{\sqrt{s}/2} \frac{dk}{\sqrt{s}} (1 - \frac{k}{\sqrt{s}}) \sigma_{BW}(s - 2k\sqrt{s}) \end{aligned} \tag{2}$$

if the compositeness of the proton is ignored. The radiation lowers the peak cross section, raises the position of the resonance peak, and introduces a high-energy tail. The function

$\beta(s, m^2)$ is the semiclassical collinear factor[4] arising from the integration over photon angle:

$$\beta(s, m^2) = \frac{2\alpha}{\pi}\Big\{\frac{s-2m^2}{\sqrt{s(s-4m^2)}}\ln\Big[\frac{s+\sqrt{s(s-4m^2)}}{s-\sqrt{s(s-4m^2)}}\Big]-1\Big\}. \tag{3}$$

The physical region for $\beta(s, m^2)$ is $s \geq 4m^2$. The asymptotic form of $\beta(s, m^2)$ is $\beta = (2\alpha/\pi)[\ln(s/m^2) - 1]$; the threshold behavior is $\beta = (2\alpha/\pi)(s - 4m^2)/3m^2$, ignoring Coulomb corrections. For $s = (3.1 \text{ GeV})^2$ and $m = m_p$, $\beta = (2\alpha/\pi)(1.23) = 0.0057$, while the asymptotic approximation gives $\beta = (2\alpha/\pi)(1.39)$. The physical difference with e^+e^- production of charmonium is due to the much smaller electron mass, where $\beta(s, m_e^2) = (2\alpha/\pi)(16.4)$ is considerably larger.

The proton's compositeness modifies the effect of radiation given in (2) by introducing anomalous static and dynamical proton properties. A multipole analysis[4] of these shows that the $dk\ k^{-1}$ term depends only on the kinematics and proton charge, while the dk term is an electric dipole term. Since the proton lacks an intrinsic electric dipole, this term as well is purely kinematical. These conclusions also follow from the soft photon theorem of Low,[5] in that the first two terms in k of bremsstrahlung are fixed by kinematics and gauge invariance only. All dependence on the proton's anomalous structure starts with the $dk\ k$ term, which, besides having terms depending only on kinematics, also includes terms depending on the proton's electric and magnetic form factors, the proton's static magnetic moment, and the proton's virtual mixing with higher states such as the $\Delta(1240)$. The virtual photon corrections involving the resonant charmonium state itself also begin at this order.

Because the radiation is produced on resonance, the photon energy $k \lesssim \Gamma_R$. Hence, the kinematical contribution to the $dk\ k$ term in (2) is of order $(\alpha/\pi)\Gamma_R^2/M_R^2$; the contribution depending on the proton structure is of order $(\alpha/\pi)\Gamma_R^2/Q_p^2$, where $Q_p \gtrsim 0.8$ GeV; and the contribution due to the charmonium structure is of order $(\alpha/\pi)\Gamma_R^2/Q_c^2$, where $Q_c \gtrsim 0.4$ GeV, all relative to the uncorrected cross section. These corrections are all completely negligible. The dk term, a correction of order $(\alpha/\pi)\Gamma_R/M_R$, in the e^+e^- case produces the so-called "hard-photon" or radiative tail in the cross section in the region two or more widths above the resonance peak.[2,3] The hard-photon effect is negligible at the peak, but since the cross section drops rapidly for s above the peak position, the hard-photon tail is visible in that region. In the $p\bar{p}$ case, however, β is more than an order of magnitude smaller, and the hard-photon contribution to $\sigma(s)$ in the tail region:

$$\sigma(s)_H = \frac{-\pi\beta\Gamma_R}{\sqrt{s}}\sigma_0 \tag{4}$$

has a tiny effect (less than one percent, at most) on the lineshape.

In many cases, the charmonium resonance in question decays to charged final-state products (such as e^+e^-), requiring interference radiation and box corrections between the initial and final states. But, as is known from the case of the Z° resonance,[6] this radiation correction is of order $(\alpha/\pi)\Gamma_R^2/M_R^2$ — the longer lived the resonance, the less influence the initial state has on the final.

Thus the bremsstrahlung correction in the $p\bar{p}$ case is simpler than the e^+e^- case: only the leading $dk\ k^{-1}$ soft term is important, as long as the correct value of $\beta(s, m_p^2)$ given by (3) is used. This correction is infrared-divergent at lowest order but is modified into convergent exponentiated form once multiple soft photon emission is included:

$$\sigma(s)_S = \beta\int_0^{\sqrt{s}/2} \frac{dk}{k}\left(\frac{2k}{\sqrt{s}}\right)^\beta \sigma_{BW}(s - 2k\sqrt{s}). \tag{5}$$

This soft radiation spectrum gives rise to the corrected resonance properties found by previous authors;[2,3] i.e., the shift in peak position, the renormalization of the peak cross section, and the radiation-corrected version of the so-called "area rule," which relates the resonance width to the peak position and cross section and the measured area under the resonance curve. Because the values of β and Γ_R/M_R are so small, the peak position shift is negligible, but the radiation does affect the peak cross section and width determinations.

The Breit-Wigner $\sigma_{BW}(s)$ must also be convoluted with the beam energy resolution function. The specific form of the corrected resonances then depends on the magnitude of the beam energy resolution relative to the resonance widths.[2]

The radiation from the p and $\bar{p}$ lines is mildly collinear with the beams, emitted within average transverse angle $\theta \simeq \gamma^{-1} = 2m/\sqrt{s} \sim 35°$. For final states with detected individual photons, the form (2) is not sufficient, since the photon angular distribution is significant. It is still important to treat the (anti)protons as moderately, not extremely, relativistic.

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